

(3 hours)

Total Marks: 80

N.B.

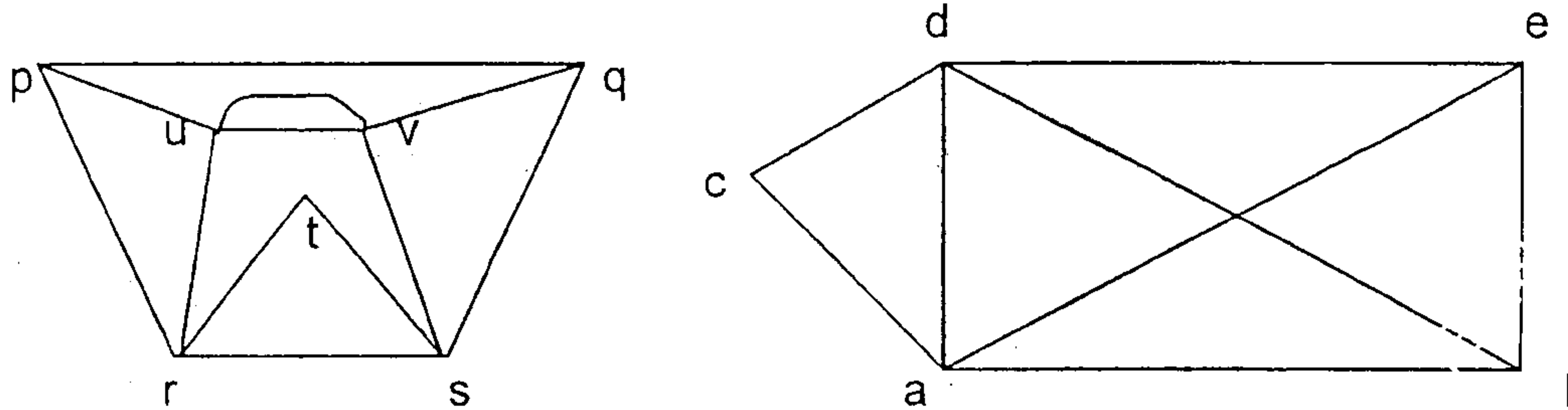
1. Question No 1 is compulsory
2. Solve any three question out of remaining five questions
3. Assumption made should be clearly stated
4. Figure to the right indicates full marks

- 1 (a) Prove that $8^n - 3^n$ is a multiple of 5 by mathematical induction, $n \geq 1$ 5
- (b) Show that if a relation on set A is transitive and irreflexive, then it is asymmetric. 5
- (c) Function $f(x) = (4x + 3)/(5x - 2)$. Find f^{-1} 5
- (d) What is the total number of vertices in a full binary tree with 20 leaves? 5
2. (a) Let $f(x) = x + 2$, $g(x) = x - 2$ and $h(x) = 3x$ for all $x \in R$. (R is the set of real number). Find i) $f \circ g \circ h$ ii) $h \circ g \circ f$ iii) $f \circ f \circ f$ 8
- (b) Let R be a relation on the set of integers Z defined by aRb if and only if $a \equiv b \pmod{5}$. Prove that R is an equivalence relation. Find Z/R . 8
- (c) Show that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ 4
- 3 (a) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,2), (2,3), (3,4), (2,1)\}$. Find the transitive closure using Warshall's algorithm. 6
- (b) Consider the lattices $L1 = \{1, 2, 4\}$, $L2 = \{1, 3, 9\}$ under divisibility. Draw the lattice $L1 \times L2$. 7
- (c) Solve the recurrence relation $a_n = -3(a_{n-1} + a_{n-2}) - a_{n-3}$ with $a_0 = 5$, $a_1 = -9$ and $a_2 = 15$ 7
- 4 (a) Show that a group G is abelian if and only if $(ab)^2 = a^2b^2$ for all $a, b \in G$ 6
- (b) Prove that the set $G = \{1, 2, 3, 4, 5, 6\}$ is an abelian group under multiplication modulo 7. 6
- (c) Find the generating function for the following series 8
- i) $\{0, 1, 2, 3, 4, \dots\}$
 - ii) $\{1, 2, 3, 4, 5, \dots\}$
 - iii) $\{2, 2, 2, 2, 2, \dots\}$
 - iv) $\{0, 0, 0, 1, 1, 1, 1, \dots\}$
- 5 (a) Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be parity check matrix. 8
- Decode the following words relative to maximum likelihood decoding function.
- i) 011001
 - ii) 101011
 - iii) 111010
 - iv) 110110

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(b) Determine the Eulerian and Hamiltonian path, if exists, in the following graphs:

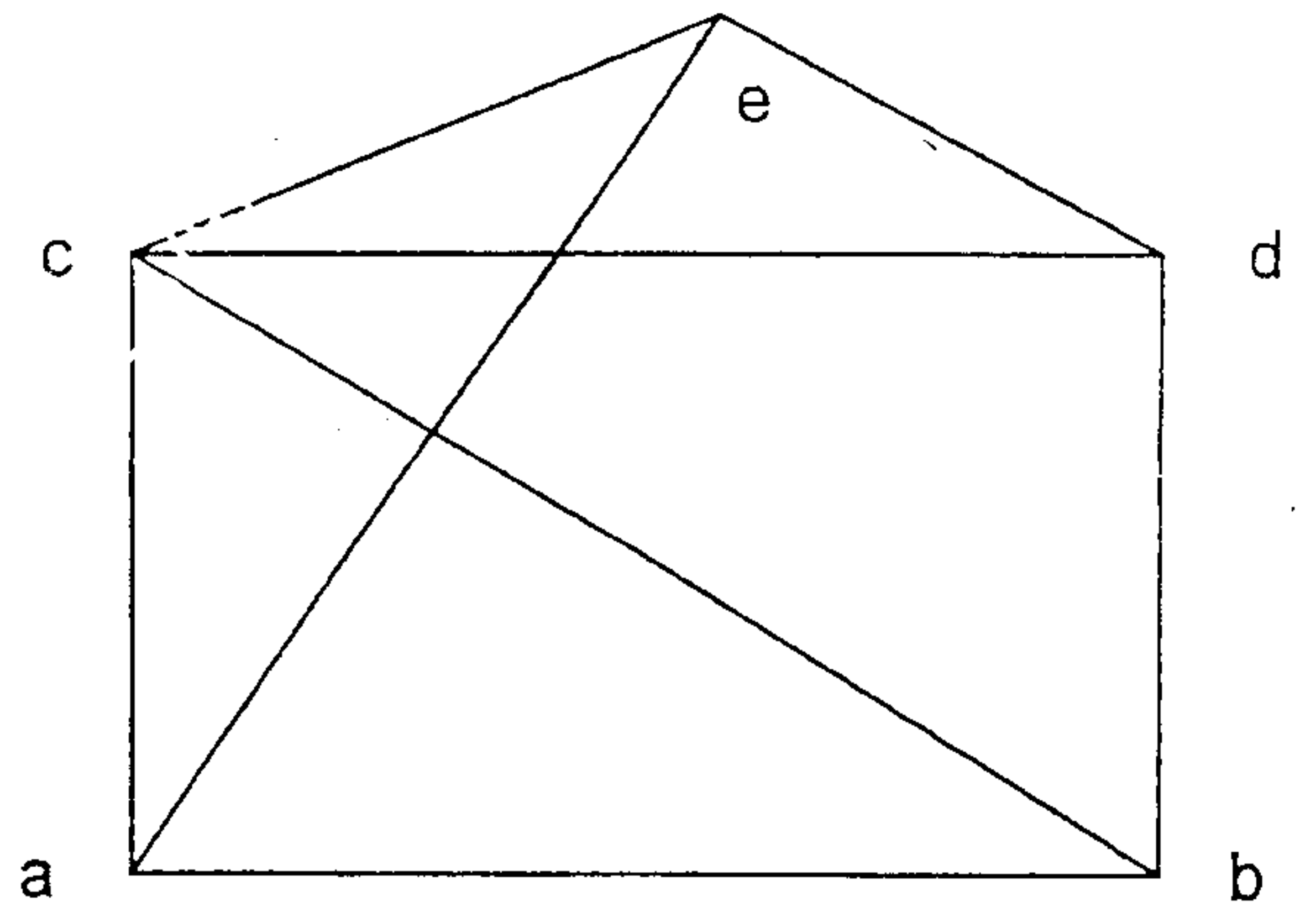
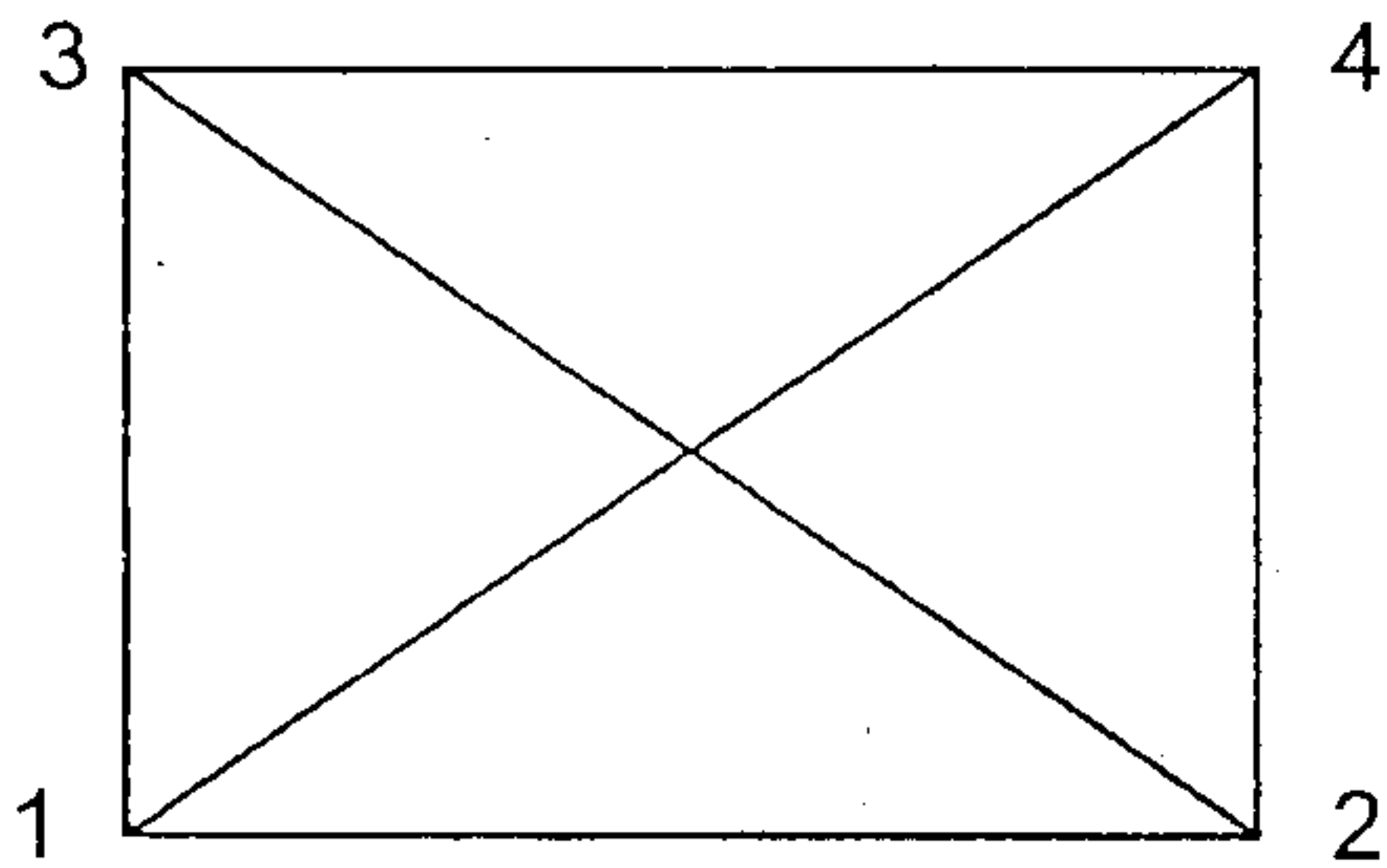
6



(c) Let G be the set of real numbers and let $a * b = ab/2$. Show that $(G, *)$ is an abelian group.

6

6 (a)



8

(b) Use the laws of logic to determine the following expression as tautology or contradiction.

6

$$[p \wedge (p \Rightarrow q)] \Rightarrow q$$

(c) Draw the Hasse Diagram of the following:

6

a) D_{105}

b) D_{72}